NONLINEAR DYNAMICS OF FLUID AND STRUCTURAL SYSTEMS

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Duke University
ABSTRACT

Fluid and structural systems and their possible interaction have a rich array of behavior being susceptible to instabilities and thus the generation of limit cycle oscillations and on occasion chaotic response. In this talk we will touch on several example including solar sails, high performance aircraft and aerodynamic decelerators from space into planetary atmospheres. Much of the talk will be devoted to the large and small scale oscillations that may appear in fluid flows and thus excite structural motion. The large scale motions include buffet in aircraft and non-synchronous vibration in jet engines, the classic case being the Von Karman vortex street. The small scale motions have as their most well known example the transition from laminar to turbulent flow. In all of the above cases a combination of theory, computation and experiment is used to understand the nonlinear dynamics of such systems.
ABSTRACT

• FLUID FORCES → STRUCTURAL DEFORMATION

• LINEAR MODELS DESCRIBE ONSET OF DYNAMIC INSTABILITIES

• NONLINEAR MODELS DESCRIBE LIMIT CYCLE OSCILLATIONS

• EXAMPLES
  • SOLAR SAIL
  • FLAPPING FLAGS
  • BUFFET
  • TURBULENCE
SEMINAR OUTLINE

SOLAR SAIL AND SOLARELASTICITY - CHAD GIBBS/NASA LANGLEY

FLAPPING FLAG (A CANTILEVERED PLATE IN AXIAL FLOW) – CHAD GIBBS/NSF

BUFFET (THE VON KARMAN VORTEX STREET REVISITED) - JEFF THOMAS/AIR FORCE

REDUCED ORDER MODELS OF THE NAVIER-STOKES EQUATIONS - MIKE BALAJEWICZ/NSF
Flutter of Thin Flexible Space Structures due to Solar Radiation

Vacuum chamber ground vibration experiment setup including instrumentation

Solar sail concept
• Working with NASA Langley to explore the dynamic coupling between large, flexible solar sails and solar radiation pressure to identify instabilities

• *Structural model*: Linear plate/membrane model. "*Fluid*” model: Solar Radiation Pressure

• Modal expansion and Lagrange’s Equations used to derive equations of motion.

• Validate the models with ground experiments in a vacuum chamber. Ground vibration experiments have already been completed
Results

Instability boundary as a function of the spin rate identified an instability at the nominal design spin rate.

Example of ground vibration experiment conducted in the NASA Langley 8-ft vacuum chamber
Conclusions and Future Work

- Initial explorations show that solarelastic phenomenon may be present in solar sails and therefore accurate models must be developed.
- Validate the model by comparing the model to other finite element models.
- Improve fidelity of the model to increase the operating conditions that can be accurately captured.
- Continue to validate the theoretical model with experimental tests whenever possible. A heliogyro demonstration mission poses a great opportunity to validate current solarelastic modeling.
Flutter Analysis of a Cantilevered Beam: Experimental Exploration of the Limit Cycle Oscillation

Theoretical prediction of the Limit Cycle Oscillation of the flapping flag system. Flow visualization (PIV) viewing window and tip motion are highlighted.
Conclusions and Future Work

- Experiment cheaply quantified the limit cycle oscillation amplitude vs. velocity providing a valuable data set for evaluating non-linear models of the aeroelastic system.

- Explore LCO for additional configurations to quantify the observed hysteresis loop as a function of parameter variation.
Conclusions and Future Work

- PIV techniques show that the flow remains attached to the flapping flag, even in a violent, large amplitude LCO.

- Future work includes investigating the wake behind the plate to explore any large wake structures that are formed during the LCO.
Computational Modeling of Unsteady Separated Incompressible Flows for an Airfoil at High Angles-of-Attack

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July 22, 2013
OVERVIEW

- Harmonic Balance Technique for Modeling Buffet and Lock-in
- Nonlinear Reduced Order Model?
Experimental Configuration

Duke Wind Tunnel
NACA 0012 Airfoil Configuration

\[ 0 \leq \alpha_0 \leq 60 \text{ Degrees}, \]
\[ 87,000 \leq Re_\infty \leq 478,000, \]
\[ M_\infty \approx 0 \]
\[ 0 \leq \alpha_1 \leq 1.5 \text{ Degree} \]
Computed High Angle-of-Attack Shedding

OVERFLOW Instantaneous Total Pressure Contours
$M_\infty \approx 0$, $Re_\infty = 100,000$

12 Degrees  20 Degrees  30 Degrees
12 Degrees  20 Degrees  30 Degrees
40 Degrees  50 Degrees  60 Degrees
Time-Domain Response

Lift Coefficient Time Histories

$M_\infty \approx 0$, $Re_\infty = 100,000$

12 Degrees

20 Degrees

30 Degrees

40 Degrees

50 Degrees

60 Degrees
Frequency Response

Fourier Transfer of Lift Coefficient Time Histories

\( M_\infty \approx 0, \quad Re_\infty = 100,000 \)

12 Degrees

20 Degrees

30 Degrees

40 Degrees

50 Degrees

60 Degrees
Effect of Turbulence Model

Lift Coefficient Time Response
\[ \alpha_0 = 40.0 \text{ Degrees}, \quad Re_\infty = 200,000, \quad \text{and} \quad M_\infty = 0.1 \]

Spalart-Almaras  Wilcox's \( k - \omega \)  Menter's SST \( k - \omega \)
Effect of Turbulence Model

Fourier Transform of Lift Coefficient Time Response
\( \alpha_0 = 40.0 \) Degrees, \( Re_\infty = 200,000 \), and \( M_\infty = 0.1 \)

Spalart-Almaras    Wilcox’s \( k - \omega \)    Menter’s SST \( k - \omega \)
Effect of Wind Tunnel Walls

$\alpha_0 = 40.0$ Degrees, $Re_\infty = 200,000$, and $M_\infty = 0.1$

Overset Computational Meshes

Instantaneous Total Pressure Contours

Frequency Response

Fourier Transformation of Lift Coefficient History

$\tilde{\omega} = \omega c/\bar{U}_\infty$

Reduced Frequency, $\tilde{\omega} = \omega c/\bar{U}_\infty$
Harmonic Balance Shedding Frequency Search Technique

Unsteady First Harmonic Unsteady Lift Phase Change Per Iteration

$\alpha_0 = 40.0$ Degrees, $Re_\infty = 200,000$, and $M_\infty = 0.1$

![Graph showing shedding frequency]

Shedding Frequency

$\bar{\omega} \approx 1.583$
Harmonic Balance Shedding Frequency Search Technique

Unsteady First Harmonic Unsteady Lift Phase Change Per Iteration

HB Solution Residual Convergence

\[ Q^n \quad (L_1 Re^n, L_1 fm^n) \quad \phi^n \quad \Delta \phi^n = \phi^{n+1} - \phi^n \]
\[ Q^{n+1} \quad (L_1 Re^{n+1}, L_1 fm^{n+1}) \quad \phi^{n+1} \quad \Delta \phi^{n+1} = \phi^{n+2} - \phi^{n+1} \]
\[ Q^{n+2} \quad (L_1 Re^{n+2}, L_1 fm^{n+2}) \quad \phi^{n+2} \quad \Delta \phi^{n+2} = \phi^{n+3} - \phi^{n+2} \]

...
Harmonic Balance Shedding Frequency Search Technique

Unsteady First Harmonic Unsteady Lift Phase Change per Iteration

\[ \alpha_0 = 40.0 \text{ Degrees}, \quad Re_\infty = 200,000, \quad \text{and} \quad M_\infty = 0.1 \]

![Graph showing shedding frequency with \( \omega = 1.583 \)]
Shedding Frequency Results Summary

\[ \alpha_0 = 40.0 \text{ Degrees}, \quad Re_\infty = 200,000, \quad \text{and} \quad M_\infty = 0.1 \]

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( \bar{\omega}_b \approx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>2.2</td>
</tr>
<tr>
<td>OVERFLOW / Isolated Airfoil / Spalart and Allmaras</td>
<td>1.5</td>
</tr>
<tr>
<td>OVERFLOW / Isolated Airfoil / Wilcox's ( k - \omega )</td>
<td>1.2</td>
</tr>
<tr>
<td>OVERFLOW / Isolated Airfoil / Menter's SST ( k - \omega )</td>
<td>1.2</td>
</tr>
<tr>
<td>OVERFLOW / Wind Tunnel Walls / Spalart and Allmaras</td>
<td>2.0</td>
</tr>
<tr>
<td>Duke HB / Isolated Airfoil / Spalart and Allmaras</td>
<td>1.6</td>
</tr>
</tbody>
</table>
Computed Lock-in Region

Harmonic Balance Method Computed Lock-in Region

\( \alpha_0 = 40.0 \) Degrees, \( Re_\infty = 200,000 \), and \( M_\infty = 0.1 \)

![Graph showing lock-in regions with symbols for locked in and not locked in]
Lock-in Region - Computed Versus Experiment

\[ \alpha_0 = 40.0 \text{ Degrees}, \quad Re_\infty = 200,000, \quad M_\infty = 0.1 \]
Lock-in

Unsteady Lift Response Response in Lock-in Region
\( \alpha_0 = 40.0 \) Degrees, \( Re_\infty = 200,000, \ M_\infty = 0.1 \)
\( f/f_b = 1, \ \alpha_1 = 1.5 \) Degree

Experiment

Computed
Lock-in

Fourier Transform of Lift Response Response in Lock-in Region
\( \alpha_0 = 40.0 \) Degrees, \( Re_\infty = 200,000 \), \( M_\infty = 0.1 \)
\( f/f_b = 1 \), \( \alpha_1 = 1.5 \) Degree

![Fourier Transformation of Lift Coefficient History](image-url)
Conclusions


2. Computational Results Are Sensitive to Turbulence Model. The Spalart-Almaras Model Yields Results Which Are in Closest Agreement with Experiment.

3. The Quantitative Agreement Between Computations and Experiments for the Reduced Frequency Is Improved When Wind Tunnel Wall Effects Are Included in the Computation.

4. Results Have Also Been Obtained for Small Oscillations of the Airfoil about a Large Fixed Angle-of-Attack and Lock-in Is Observed.

5. The Computational Results in 1-3 Were Obtained with a Time Marching Solution Method. The results in 4 Were Obtained with a Computationally Efficient Harmonic Balance Solution Method.

Reduced Order Modeling for Navier-Stokes Equations

PRESENTED AT THE SYMPOSIUM ON ADVANCES IN NONLINEAR DYNAMICS

IN HONOR OF PROFESSOR ANIL BAJAJ

PURDUE UNIVERSITY, AUGUST 10-11, 2012

EARL DOWELL AND MACIEJ BALAJEWICZ, DUKE UNIVERSITY
A New Approach to Model Order Reduction of the Navier-Stokes Equations

Ph.D. Dissertation Defense

Maciej Balajewicz
Mechanical Engineering and Materials Science, Duke University

July 23, 2012
Outline

- Introduction and motivation
  - General properties of turbulent flow
  - Projection-based Model Order Reduction (MOR)
  - Prototypical example: lid-driven cavity
- Instability mechanism
- Proposed new approach
- Results
- Conclusions and future work
Outline

• Introduction and motivation
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Turbulence is a flow regime characterized by a chaotic broad spectrum response in both space and time.

Fig 1. Sketch of turbulence, Leonardo da Vinci¹ ca. 1510

Introduction and motivation

— Projection-based Model Order Reduction (MOR)

\[ \frac{\partial}{\partial t} u(t, x) = F(u(t, x)) \]

• **Step 1:** Spectral discretization

\[ u(t, x) \approx u^{[1,\ldots,n]} = \sum_{i=1}^{n} a_i(t)u_i(x) \]

• **Step 2:** Proper Orthogonal Decomposition

  – Given a solution, \( \mathbf{u}_s(t, x) \), find basis functions such that

\[ \arg \min_{a_i, u_i} \left\| \mathbf{u}_s(t, x) - \sum_{i=1}^{n} a_i(t)u_i(x) \right\|_{\Omega,T} \]

• **Step 3:** Projection using the first \( n \) POD basis functions

\[ \left\langle \mathbf{u}_i(x), \frac{\partial}{\partial t} u^{[1,\ldots,n]} \right\rangle_{\Omega} = \left\langle \mathbf{u}_i(x), F(u^{[1,\ldots,n]}) \right\rangle_{\Omega} \quad i = 1, \ldots, n \]
Introduction and motivation
— Projection-based Model Order Reduction (MOR)

\[ \frac{\partial}{\partial t} u(t, x) = F(u(t, x)) \]

\[ \frac{d}{dt} a_i(t) = f_i(a_i(t)) \quad i = 1, \ldots, n \]
Introduction and motivation

— Prototypical example: lid-driven cavity

- Two-dimensional, incompressible flow inside a square cavity driven by a prescribed lid velocity

Fig 2. Regularized lid-driven cavity
Introduction and motivation

— Prototypical example: lid-driven cavity

**Fig 3.** DNS of the lid-driven cavity at $Re = 3 \times 10^4$
Introduction and motivation

— Prototypical example: lid-driven cavity

Fig 4. POD spatial basis functions of the lid-driven cavity at $Re = 3 \times 10^4$
Fig 5. Evolution of the vorticity field of the lid-driven cavity
Introduction and motivation

— Prototypical example: lid-driven cavity

- Turbulent kinetic energy

\[ E(t) = \frac{1}{2} \int_{\Omega} |u(t, x)|^2 \, dx \]

Fig 6. Evolution of the turbulent kinetic energy of the lid-driven cavity
Introduction and motivation

— Prototypical example: lid-driven cavity

Fig 7. Evolution of the vorticity field of the lid-driven cavity

a) DNS

b) Optimal reconstruction using $n = 200$ POD basis functions

c) ROM, $n = 200$
Introduction and motivation

— Prototypical example: lid-driven cavity

Fig 8. Evolution of the turbulent kinetic energy of the lid-driven cavity
Outline

- Introduction and motivation
  - General properties of turbulent flow
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Instability mechanism

— Turbulence is a multi-scale phenomenon

“Big whorls have little whorls that feed on their velocity,
And little whorls have lesser whorls and so on to viscosity”\(^2\)

\(^2\)L. F. Richardson, *Weather prediction by numerical process* (University press, Cambridge, 1922)
Instability mechanism

— Turbulence is a multi-scale phenomenon

Instability mechanism

— POD, by definition, is biased towards energy production
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Proposed new approach

— Abstract formulation

Derive a new set of basis functions that resolve a more balanced distribution of kinetic energy production and dissipation

• Can be formulated as a minimization problem

\[
\arg \min_{a'_i, u'_i} \left\| u_s(t, x) - \sum_{i=1}^{n} a'_i(t)u'_i(x) \right\|_{\Omega,T}
\]

s.t. More dissipative scales are resolved
Proposed new approach

— Numerical implementation

• Assume the $n$ new basis functions are spanned by $N$ ($N>n$) POD basis functions

$$u'_i(x) = \sum_{j=1}^{N} T_{ji} u_i(x) \quad T \in \mathbb{R}^{N \times n}$$

• Projection using these new basis results in

$$\ddot{a}'_i(t) = f_i (a'_i(t), \mathbf{T}) \quad i = 1, \ldots, n$$
Proposed new approach

— Numerical implementation

• For flows with steady Dirichlet, free-stream, periodic and convective boundary conditions, the minimization problem can be expressed as follows

\[
\arg\min_{T \in \mathbb{R}^{N \times n}} \sum_{i=1}^{N} \lambda_i - \sum_{i=1}^{n} \left( T^T \text{diag}\{\lambda\} T \right)_{ii}
\]

s.t  \( (1) \ T^T T = I_{n \times n} \)

\( (2) \ \sum_{i=1}^{n} \left( T^T D T \circ T^T \text{diag}\{\lambda\} T \right)_{ii} = -\varepsilon \)

• Where \( \varepsilon \) quantifies the energy production/dissipation of the new basis functions
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Fig 10. Evolution of the vorticity field of the lid-driven cavity

a) DNS  
b) ROM, $n = 10$ (Standard)  
c) ROM, $n = 10$ (Stabilized)
Results

Fig 11. Evolution of the turbulent kinetic energy of the lid-driven cavity
**Fig 12.** Net kinetic energy production/dissipation vs. ROM order
• The new basis functions are very similar to the POD basis functions

Fig 13. First spatial basis function of the lid-driven cavity
Results

- In other words, the new basis functions

\[ u'_i(x) = \sum_{j=1}^{N} T_{ji} u_i(x) \quad T \in \mathbb{R}^{N \times n} \]

are defined by a transformation matrix of the form

\[ T \approx \begin{bmatrix} I_{n \times n} \\ 0_{(N-n) \times n} \end{bmatrix} + \Delta_{N \times n} \]

where \( I \) and \( 0 \) are the identity and null matrices respectively and \( \Delta \) is matrix whose entries are all less than one.
Results

- For example, for $n = 2$ and $N = 10$

\[
T = \begin{bmatrix}
0.9993 & 0.0001 \\
0.0001 & 0.9996 \\
0.0044 & 0.0464 \\
-0.0056 & -0.0084 \\
0.0563 & -0.0799 \\
-0.0640 & -0.0317 \\
0.0199 & -0.0138 \\
0.0404 & -0.0368 \\
-0.0751 & 0.0344 \\
0.0783 & 0.0159
\end{bmatrix}
\]
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Conclusions

• A new approach to Model Order Reduction of the Navier-Stokes equations was proposed.

• The method provides spatial basis functions different from the usual proper orthogonal decomposition (POD) basis functions.

• These new basis functions resolve a greater range of physical scales of the turbulent fluid flow compared to the POD basis functions that are, by construction, biased toward the large, energy containing scales.

• When these basis functions are used in a Galerkin projection of the Navier-Stokes equations, more stable and accurate ROMs are derived.
Future work

- The appropriate balance between kinetic energy production and dissipation, $\varepsilon$ is not known a priori.

$$\arg\min_{T \in \mathbb{R}^{N \times n}} \sum_{i=1}^{N} \lambda_i - \sum_{i=1}^{n} \left( T^T \text{diag}\{\lambda\} T \right)_{ii}$$

s.t.  
(1) $T^T T = I_{n \times n}$

(2) $\sum_{i=1}^{n} \left( T^T D T \circ T^T \text{diag}\{\lambda\} T \right)_{ii} = -\varepsilon$
Thank you